

Temporal Memory Index: A Novel Feature for Handwriting Analysis in Developmental Dysgraphia

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Abstract— Digital handwriting analysis offers a promising, non-invasive approach to assessing neurodevelopmental disorders like developmental dysgraphia (DD). Existing methods, while helpful, may not fully capture the subtle temporal dynamics and memory effects present in handwriting. This study introduces the Temporal Memory Index (TMI), a novel metric based on fractional order derivatives (FDs), designed to quantify these temporal dependencies. Using data from a fast Archimedean spiral task, we evaluated TMI's ability to differentiate between children with DD (n=155) and intact children (n=114). We extracted traditional kinematic features, FD features, and the proposed TMI features. While statistical tests showed no significant individual feature differences between groups, classification analysis (XGBoost) resulted in an increase in the specificity. Although modest in terms of overall classification accuracy, these preliminary findings suggest that TMI captures information about temporal dynamics in handwriting and warrants further investigation as a potential marker for DD.

Keywords— *temporal memory index, fractional calculus, developmental dysgraphia, graphomotor disabilities, online handwriting,*

I. INTRODUCTION

Handwriting is a complex motor skill that reflects underlying neurological processes, making its analysis a valuable tool for assessing neurodevelopmental disorders. Digital handwriting analysis, in particular, offers a non-invasive and objective means of capturing subtle motor control deficits that might not be apparent through traditional observation [1].

Developmental dysgraphia (DD), a specific learning disability affecting handwriting fluency and legibility, presents a significant challenge for affected children [2]. Early and accurate identification of DD is crucial for implementing appropriate interventions. While traditional kinematic features (e.g., velocity, acceleration) and, more recently, fractional derivatives (FDs) have been used to analyze handwriting in DD [3], [4], these approaches may not fully capture the subtle temporal dynamics and memory effects inherent in the handwriting process. These temporal dependencies, reflecting the influence of past movements on current motor output, could be key to differentiating DD from typical handwriting. This limitation highlights the need for new analytical approaches that explicitly consider the temporal dependencies within handwriting movements.

This study introduces the Temporal Memory Index (TMI), a novel metric based on the difference between fractional derivatives of different orders, to quantify the degree of temporal memory in handwriting signals. Intuitively, TMI reflects the influence of past movements on the current state of a writer's pen trajectory, capturing information not readily available from instantaneous kinematic measures or even single-order FDs. We hypothesize that TMI could provide a more sensitive marker for distinguishing children with DD from intact children, potentially revealing subtle motor control differences not apparent with existing methods.

II. MATERIALS AND METHODS

A. Temporal Memory Index (TMI) by Fractional Order Derivatives

Fractional calculus (FC) extends the concept of differentiation and integration to non-integer orders, providing a powerful tool for modeling systems with memory and long-range dependence [5]. Unlike integer-order derivatives, which are local operators, fractional derivatives incorporate information from the history of a signal. Our previous work has demonstrated the ability of FDs to enhance the differentiation of handwriting characteristics in neurodegenerative disorders [4] and children with DD [3], [6], suggesting their sensitivity to subtle nuances in motor control and coordination.

The Caputo fractional derivative is a commonly used definition appropriate for initial value problems. It is defined as [7]:

$${}^C D^\alpha y(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} y'(\tau) d\tau, \quad (1)$$

where ${}^C D^\alpha y(t)$ denotes the Caputo derivatives of order α of the function $y(t)$, Γ is the gamma function, and $n-1 < \alpha \leq n$, $n \in \mathbb{N}$, $t > 0$.

We propose a novel temporal memory index (TMI) based on the difference between fractional derivatives calculated with different orders to quantify the degree of temporal memory in handwriting signals. To compute TMI, we first define the alpha-difference vector (ADV) as:

$$\text{ADV}(t) = D^{\alpha_2} f(t) - D^{\alpha_1} f(t), \quad (2)$$

where $\alpha_2 > \alpha_1$. The ADV captures the difference in past influence on the signal, as represented by the two fractional derivatives. A larger difference indicates a stronger memory effect.

The TMI is then defined as the mean absolute value of the ADV over a given time interval:

$$\text{TMI} = \frac{1}{T} \int_0^T |\text{ADV}(t)| dt \approx \frac{1}{N} \sum_{i=1}^N |\text{ADV}(t_i)|, \quad (3)$$

where T is the time interval length, and N is the number of the samples.

To validate the proposed TMI and guide the selection of appropriate FDs (α_1, α_2) for the handwriting analysis, we conducted preliminary experiments using fractional Brownian motion (fBm) signals [8]. The fBm provides a well-established model for processes with controlled long-range dependence, characterized by the Hurst exponent (H), where $0 < H < 1$. Values of $H < 0.5$ indicate anti-persistence (short-term memory), $H = 0.5$ corresponds to standard Brownian motion (no long-range memory), and $H > 0.5$ indicates persistence (long-term memory). We generated fBm signals using the Davies-Harte method with three representative Hurst exponents: $H = 0.1$ (anti-persistent), $H = 0.5$ (no long-range memory), and $H = 0.8$ (persistent). For each H value, we calculated the TMI using three (α_1, α_2) pairs: (0.1, 0.5), (0.1, 0.9), and (0.5, 0.9). These pairs were chosen to represent varying degrees of difference between the FDs, allowing us to assess the TMI's sensitivity to this difference. The signals' lengths were set to 1000, with dt set to 0.01.

Figure 1 shows the calculated TMI values for the different H and (α_1, α_2) combinations. As expected, the TMI is lowest for $H = 0.5$, corresponding to standard Brownian motion with no long-range dependence. The TMI is significantly higher for $H = 0.1$, reflecting the anti-persistent nature of the signal. The large difference between α_1 and α_2 captures the discrepancy between short-term fluctuations and a longer-term "memory" that is not truly present. The TMI is also elevated for $H = 0.8$ compared to $H = 0.5$ but lower than for $H = 0.1$. This indicates that while the long-range dependence influences both fractional derivatives, the smaller α_1 still captures more short-term variations, leading to a non-zero difference. Furthermore, for a fixed H , the TMI is consistently higher for the (0.1, 0.9) pair, representing the largest difference in memory lengths considered. This highlights the influence of the past on the signal.

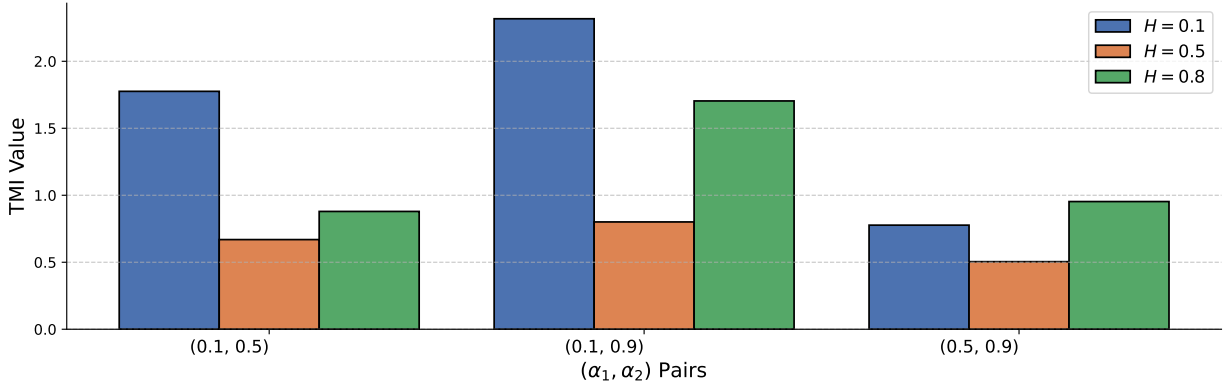


Fig. 1. Temporal Memory Index (TMI) values for different Hurst exponent (H) and (α_1, α_2) pairs.

B. Database

We enrolled 269 children from Czech primary schools, stratified into two groups: intact children (IC, 114) and those with developmental dysgraphia (DD, 155), as diagnosed by a special education needs teacher. Children were instructed to complete a fast Archimedean spiral on a Wacom Cintiq 16 (DTK1660K0B) display tablet during data acquisition. The fast spiral task was chosen for this exploratory study to examine potential differences in motor control between the groups. Informed consent was obtained from all parents, and the study was approved by the Ethics Committee of the Institute of Psychology at the Czech Academy of Sciences, adhering to the American Psychological Association's Ethical Principles [9].

C. Feature Extraction

We analyzed only the on-surface movement and computed the basic kinematic features, velocity, acceleration, and jerk, along with their horizontal (x) and vertical (y) components. To capture temporal memory characteristics, we then calculated FDs of the same kinematic features using the Caputo definition (Equation 1) and the orders $\alpha = \{0.1, 0.3, 0.5, 0.7, 0.9\}$. These values were chosen to span a range of memory lengths, from short-term to longer-term influences.

We computed the novel proposed TMI using all pairwise combinations of those above α values. This allowed us to explore the sensitivity of TMI to different memory length comparisons. All features were calculated per stroke, and the mean,

standard deviation, and median were computed to get scalar values, yielding 18 baseline kinematic features, 90 FD features, and 180 TMI features (288 total).

D. Statistical Analysis and Machine Learning

Prior to statistical analysis, we considered grade and gender as confounding factors and removed their effects using linear regression [10]. Subsequently, we calculated Spearman's correlation (with False Discovery Rate (FDR) correction for multiple comparisons) to explore the relationships between handwriting features and clinical group (IC/DD). We assessed the normality of each feature distribution using the Shapiro-Wilk test to compare feature distributions between the DD and IC groups, and we employed t-tests (significance threshold: adjusted p-value < 0.05).

Finally, we performed a classification analysis using the XGBoost algorithm to evaluate the discriminative power of the features. We used a 10-fold cross-validation with five repetitions. With Matthew's Correlation Coefficient (MCC) as the primary optimization metric, a Bayesian optimization strategy was employed to fine-tune the hyperparameters. The model's performance was evaluated using balanced accuracy (BACC), sensitivity (SEN), specificity (SPE), and MCC. We assessed feature importance to quantify each feature's contribution to the classification.

III. RESULTS

Based on the results of correlation analysis, various features derived from the TMI showed a positive correlation with the clinical status. However, after adjusting the p-values, only one TMI feature (median TMI of velocity ($\alpha_1=0.5$, $\alpha_2=0.7$) exhibited a borderline significant correlation ($p = 0.058$). In contrast, nine FD-based features derived from the velocity (predominantly horizontal velocity) showed statistically significant positive correlations with the clinical status.

Following the correlation analysis, we conducted t-tests between the IC and DD groups. After the p-value adjustment, none of the individual features showed a statistically significant difference between the two groups.

TABLE I. RESULTS OF THE CLASSIFICATION PERFORMANCE

Feature Set	Balanced Accuracy	Sensitivity	Specificity	MCC
All features (baseline + FD + TMI)	0.579 ± 0.070	0.854 ± 0.099	0.304 ± 0.120	0.201 ± 0.177
Baseline + FD (without TMI)	0.580 ± 0.077	0.717 ± 0.125	0.442 ± 0.115	0.171 ± 0.163

¹ FD – Fractional order derivative; TMI – Temporal Memory Index.

Table I presents the classification results using XGBoost. Classification performance was modest for both feature sets. BACC was slightly above chance (0.5) for both all features (0.579 ± 0.070) and baseline + FD features (0.580 ± 0.077). Excluding TMI features resulted in an increase in specificity (from 0.304 ± 0.120 to 0.442 ± 0.115) and a decrease in sensitivity (from 0.854 ± 0.099 to 0.717 ± 0.125).

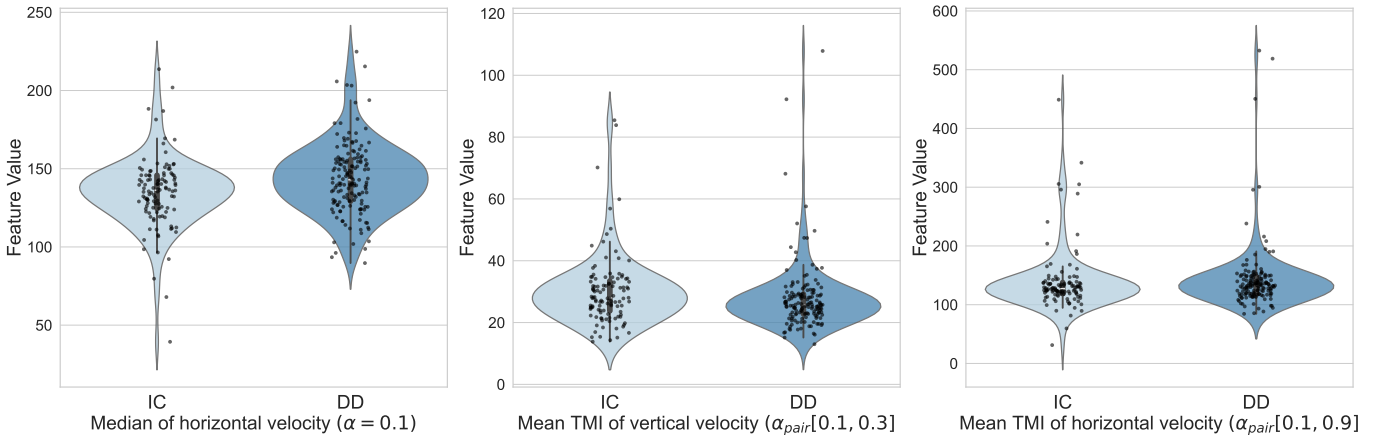


Fig. 2. Distribution of selected features across IC and DD groups.

Figure 2 illustrates the distributions of three representative features: a non-TMI feature (median horizontal velocity) and two TMI features (mean TMI of vertical and horizontal velocities). These distributions are examined to provide a visual context for the statistical and classification results.

IV. DISCUSSION AND CONCLUSION

This study introduced a novel Temporal Memory Index (TMI) based on FDs to quantify temporal memory effects in handwriting signals, specifically in the context of differentiating children with DD from IC. Our primary goal was to explore the potential of TMI as a sensitive marker for subtle motor control differences associated with DD.

The initial correlation analysis and t-test findings did not uncover exceptionally significant results. Nevertheless, this aligns with our previous research highlighting the importance of advanced and more robust parameters [3]. The inherent variability within the DD group and the relatively small sample size in each group after stratification likely contributed to the lack of statistically significant differences at the individual feature level [1]. However, it is crucial to recognize that our results are preliminary and require further validation with larger, more diverse cohorts. Moreover, the performance of TMI is comparable to that of the FD and could deserve further investigation.

Figure 2 presents the distributions of representative TMI and non-TMI features, visually demonstrating the complexity of differentiating between IC and DD groups based on single features. The considerable overlap in distributions across all features underscores the multifaceted nature of dysgraphia and the need for multifaceted analytical approaches.

The classification analysis, using XGBoost, further supports this notion. While the overall classification performance was modest, including TMI features, it did not substantially degrade performance compared to only baseline and FD features. The decrease in sensitivity when TMI features were excluded, coupled with an increase in specificity, suggests that TMI might contribute to a more nuanced understanding of the underlying differences, even if it doesn't dramatically improve overall classification accuracy. This pattern—modest overall performance but hints of differential contributions from different feature sets is typical in early-stage research exploring complex neurological conditions [6]. The temporal memory effect, as captured by TMI, might reflect subtle timing and coordination deficits that are not fully captured by traditional kinematic or even FD-based features.

This study has several limitations. First, we focused on a single handwriting task (fast Archimedean spiral). Future research should explore the performance of TMI across a broader range of handwriting tasks, varying in complexity and motor demands, to better understand its generalizability. Second, our TMI calculation was relatively simple, based on the mean absolute difference between FDs. More sophisticated approaches, potentially incorporating weighting schemes or different distance metrics, could enhance the sensitivity of TMI. Third, this study focused exclusively on DD. Investigating the behavior of TMI in other neurodevelopmental and neurodegenerative disorders characterized by motor control deficits (e.g., Parkinson's disease) is crucial to determine its broader applicability.

In conclusion, this work introduces the Temporal Memory Index as a potentially valuable tool for analyzing temporal memory effects in handwriting signals.

ACKNOWLEDGMENT

This work was supported by the Czech Science Foundation, project no. GN23-06074O (Research of advanced methods of graphomotor disabilities analysis based on fractional calculus), by the project Mediated Society (MEDIS:ON) CZ.02.01.01/00/23_025/0008713 co-financed/supported by the European Union and Czech Republic, and project no. 25-15668S supported by the Czech Science Foundation.

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